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Evaluation of the results of multi-attribute group decision-making with linguistic information

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ABSTRACT

Multi-attribute group decision-making (MAGDM) has received increasing attentions in both engineering and economy fields. Correspondingly, many valuable methods have been developed to solve various MAGDM problems, but relatively, very few research results focus on the evaluation of the effect of MAGDM. In this paper, based on the existing MAGDM methods with linguistic information, three key evaluation indices, consistency, closeness and uniformity, are proposed to measure the results of MAGDM from different aspects. By comparing the individual overall preference values with the collective ones, the three indices cannot only provide a reference for judging the decision-making effect of each decision maker, but also reflect the effect of group decision-making to a certain extent. The practicality and effectiveness of the proposed method are shown by two heuristic examples. Furthermore, the proposed method will be helpful for setting and adjusting the weights of both attributes and decision makers, as well as for selecting and comparing various aggregation operators and methods in dynamic or interactive group decision-making.

1. Introduction

In general, group decision-making (GDM) problem can be defined as a decision problem with several alternatives and decision makers (DMs) that try to obtain the best solution(s) taking into account their opinions or preferences. GDM has become a hot topic in decision science and attracts broad studies from both theoretical and applied points of view [1,2]. Linguistic preference relations are usually used by DMs to express their linguistic preference information based on pairwise comparisons of alternatives. Existing decision models used to deal with linguistic GDM problems can be divided into four categories [3]: approximate model based on extension principle [4–6]; ordered language model [7–10]; 2-tuple model [11–15] and the model that computes with words directly [16–21]. Compared with the former three models, the final model cannot only avoid losing any linguistic information, but also is straightforward and very convenient in calculation, and thus, is more practical in actual applications.

As an important research topic, GDM under linguistic preference relations has received a lot of attentions from researchers. In actual decision-making problems, due to various reasons, the linguistic evaluation information given by DMs is often incomplete. By estimating the missing information based on the additive consistency property, Cabrerizo [11] and Alonso [12] studied the selection process of alternatives in GDM with incomplete two-tuple fuzzy linguistic preference relations. Xu [18,19,22] also made an in-depth study of the GDM with incomplete linguistic preference relations.

In addition, for the GDM problems with unbalanced fuzzy linguistic information and the GDM problems with incomplete information in an unbalanced fuzzy linguistic context, Cabrerizo [13,14] presented the corresponding consensus models, respectively. For the GDM problems under multigranular linguistic information, Mata [23] proposed an adaptive consensus support system model to reduce the number of consensus rounds. Xu [16,17] made a series of studies of GDM with uncertain linguistic preference relations. Perez [24,25] implemented a prototype of a decision support system model for GDM problems based on dynamic sets of alternatives where mobile devices are used by experts to provide their preferences at anywhere and anytime.

Most decision-making problems involve multiple criteria and correspondingly different multi-criteria decision-making (MCDM) methods have been proposed [26–28]. Hwang and Yoon [29] clearly categorized MCDM into two types, i.e., multi-attribute decision-making (MADM) and multi-objective decision-making (MODM) in 1981. As an important branch of GDM problems,

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multi-attribute group decision-making (MAGDM) is commonly encountered in the real world and plays a key role especially in engineering and economy fields [30–32]. The definition of MAGDM is described specifically as follows: multiple DMs make judgments or evaluations by virtue of respective knowledge, experience and preference for a decision space (i.e., a finite set of alternatives) under multiple attributes to rank all the alternatives or give evaluation information of each alternative, and then decision results from each DM are aggregated to form an overall ranking result for all the alternatives. Generally, the process of MAGDM consists of five phases: construction of evaluation system, setting of the weights and values of the attributes, normalization of decision matrix, determination of the weights of DMs, and the overall ranking.

The existing methods for MAGDM can be roughly divided into the following categories, such as methods based on fuzzy preference [33], methods based on dempster-shafer theory [34], methods based on entropy theory, methods based on TOPSIS (technique for order performance by similarity to ideal solution) and its extended forms [35], methods based on linear programming [36,37], methods based on nonlinear programming [38], methods based on rough set theory [39], methods based on cluster analysis [40], as well as methods based on various kinds of extended weighted aggregation operators [41] and so on.

Preference information in MAGDM, commonly given by the form of decision matrix, is classified by the accuracy into two styles, namely, certain information and uncertain information. Certain information includes integer, ordinal, utility value and so on. Uncertain information mainly includes rough information, fuzzy information (such as triangular fuzzy number, trapezoidal fuzzy number, intuitionistic fuzzy number, interval number, linguistic preference information, etc.) and stochastic information (such as probability preference information).

Since MAGDM problems with linguistic preference information have a wide application background in practice, studies of both theory and application of linguistic MAGDM problems have received extensive attentions [21,38,42–44]. People usually combine MAGDM methods with the aforementioned four decision models to deal with linguistic MAGDM problems. Wei [45] proposed two extended 2-tuple aggregation operators, based on which a method for linguistic MAGDM was presented. Wu [38] put forward a maximizing deviation method based on linguistic weighted arithmetic averaging (LWAA) operator and non-linear optimization. Boran [46] combined TOPSIS method with intuitionistic fuzzy set to deal with linguistic MAGDM problems based on intuitionistic fuzzy weighted averaging operator. Xu [47] proposed a linguistic hybrid averaging (LHAA) operator and studied some desirable properties of the LHAA operator, then developed a practical approach to MAGDM under linguistic environment. Chen [48] presented an interval type-2 fuzzy TOPSIS method to handle fuzzy linguistic MAGDM problems. Hatami-Marbini [49] proposed an alternative fuzzy outranking method by extending the ELECTRE I (elimination et choice translating reality) method to take into account the uncertain, imprecise and linguistic assessments provided by a group of DMs. In addition, some researchers [50,51] also transformed linguistic information into fuzzy numbers (such as trapezoidal fuzzy numbers or interval-valued trapezoidal fuzzy numbers), and proposed other linguistic GDM methods by processing the above fuzzy numbers.

Correspondingly, the MAGDM problems with uncertain linguistic information have also attracted scholars' attentions. Xu [10,20] defined some uncertain linguistic aggregation operators to solve the GDM problems with uncertain linguistic information. However, during the evaluation process DMs tend to choose languages with different numbers of linguistic phrases (i.e., multi-granularity) according to their preferences to evaluate all the alternatives. Therefore, a number of studies have recently focused on the GDM problems based on multi-granularity linguistic evaluation information. Herrera [52] and Wang [53] transformed linguistic information into triangular fuzzy numbers and studied the method of GDM with multi-granularity linguistic evaluation information. By transforming multi-granularity uncertain linguistic terms into trapezoidal fuzzy numbers, Fan [54] proposed a GDM method based on the extended TOPSIS method. Xu [55] presented a uniform approach based on linguistic evaluation scale by introducing the concepts of virtual term and virtual term index, and then proposed a method based on the term indices for GDM problems with multi-granularity linguistic information by defining the additive weighted mean (AWM) operator and the hybrid aggregation (HA) operator.

The MAGDM approaches mentioned in the above literatures just unify the multigranular linguistic information based on balanced linguistic label sets whose linguistic labels are uniformly and symmetrically distributed. But in many real-life situations, the unbalanced linguistic information appears due to the nature of the linguistic variables used in the problems [56]. Based on some unbalanced linguistic label sets and some transformation functions and by using the uncertain linguistic weighted averaging operator, Xu [56] defined two similarity measures and developed an interactive approach to MAGDM with multigranular unbalanced and uncertain linguistic information. Yu [57] defined some transformation relationships among multigranular linguistic labels (TRMLLs) to unify the unbalanced linguistic labels with different granularities into a certain unbalanced linguistic label set with fixed granularity.

For the multi-period MAGDM problems where all decision information is expressed by decision-makers in multiplicative linguistic labels at different periods, by introducing a dynamic linguistic weighted geometric (DLWG) operator and using the minimum variability model to derive the time series weights associated with the DLWG operator, Xu [58] developed an approach to multi-period MAGDM under linguistic assessments so as to derive the final ranking of alternatives, and extended the above results to uncertain linguistic environments.

With the development of linguistic preference information from complete to incomplete, certain to uncertain, single granularity to multi-granularity, balanced to unbalanced, static to dynamic, theoretical studies of linguistic MAGDM methods are increasingly rich and perfect. However, studies of other aspects are relatively less, such as evaluation and comparison of the effect of GDM, how to reflect individual preference and how to determine the reflect degree of individual preference in the collective preference. To this end, this paper proposes three indices, i.e., consistency, closeness and uniformity to evaluate the decision-making effect of DM for the existing linguistic MAGDM methods. After the individual overall preference values and the collective overall preference values are obtained in form of linguistic terms by certain existing linguistic MAGDM methods, we firstly transform them into corresponding term indices, and then compute consistency, closeness and uniformity of the individual overall preference values with respect to the collective ones based on the above term indices. Furthermore, by using the known weights of DMs we can get the weighted average of the values on each index of each DM, respectively, thus we can define the concepts of total consistency, total closeness and total uniformity to evaluate the effect of GDM. The proposed method cannot only provide a reference for judging the decision-making effect of DM, but also reflect the effect of GDM to a certain extent.

The aim of this paper is to develop some indices to evaluate the results of MAGDM with linguistic information. The rest of the paper is organized as follows. Some basic concepts of MAGDM with linguistic information are briefly reviewed in Section 2. In Section 3, based on term indices, we propose three indices, namely, consistency, closeness and uniformity, to evaluate the decision-making effect of DM in MAGDM. Furthermore, we define the concepts of total consistency, total closeness and total uniformity to evaluate the effect of GDM. In Section 4, we illustrate the practicality and effectiveness of the method proposed in this paper by analyzing two typical examples of MAGDM with linguistic information. Section 5 concludes this paper with remarks.

2. Problem description on MAGDM with linguistic information

Before going into detail, we first introduce some basic definitions whichh will be used through the rest of this paper. Let $X=\{x_1,x_2,\ldots,x_n\}$ be a discrete set of alternatives, $G=\{G_1,G_2,\ldots,G_m\}$ be the set of attributes and $\omega=\{\omega_1,\omega_2,\ldots,\omega_m\}^T$ be the weight vector of attributes where $\omega_j\geq 0$, $\sum_{j=1}^m\omega_j=1$. Let $D=\{d_1,d_2,\ldots,d_t\}$ be the set of DMs, and $v=\{v_1,v_2,\ldots,v_t\}^T$ be the weight vector of DMs. S is a finite and totally ordered discrete term set whose cardinality value is an odd number. $P=\{P^1,P^2,\ldots,P^t\}$ is the set of decision matrices of t DMs where $P^k=(P^k_{ij})_{n\times m}$, $P^k_{ij}\in S$ is a preference value in the form of linguistic variable, given by the DM $d_k\in D$ for the alternative $x_i\in X$ with respect to the attribute $G_j\in G$ where $1\leq k\leq t, 1\leq i\leq n, 1\leq j\leq m$.

For a MAGDM with linguistic information, the discrete term set can be roughly divided into the following two forms:

(i) The subscript of the linguistic term included in the discrete term set is a non-negative integer.

$$S=\{s_\alpha|\alpha=0,1,\ldots,l\},$$

where s_α represents a possible value for a linguistic variable, called linguistic term. Specially, s_0 and s_l are the lower and upper limits, respectively, l is an even number, and S must have the following characteristics [43,59]:

(1) The set is ordered: $s_\alpha\geq s_\beta$ if $\alpha\geq\beta$;
(2) There is the negation operator: $neg(s_\alpha)=s_\beta$ such that $\beta=l-\alpha$;
(3) Max operator: $max(s_\alpha,s_\beta)=s_\alpha$ if $s_\alpha\geq s_\beta$;
(4) Min operator: $min(s_\alpha,s_\beta)=s_\alpha$ if $s_\alpha\leq s_\beta$.

For example, a set of seven terms S could be

$$S=\{s_0=extrimly\ poor, s_1=very\ poor,\ s_2=poor,\ s_3=fair,\ s_4=good,\ s_5=very\ good,\ s_6=extrimly\ good\}.$$

In the integrating process of decision-making information, the integration results often do not match the elements in S. In order to facilitate computation and avoid the loss of decision-making information, the discrete term set S can be extended to a continuous term set [60] $\overline{S}=\{s_\alpha|s_0\leq s_\alpha\leq s_l,\alpha\in[0,l]\}$ whose elements also meet all the characteristics above. If $s_\alpha\in S$, we call s_α the original term and α the original term index; otherwise, we call s_α the virtual term and α the virtual term index. In general, DM uses the original linguistic terms to evaluate alternatives, and the virtual linguistic terms can only appear during the operation.

(ii) The subscript of the linguistic term included in the discrete term set is zero-centrosymmetric.

$$S'=\{s_\alpha|\alpha=-l,\ldots,-1,0,1,\ldots,l\},$$

where s_α represents a possible value for a linguistic variable, called linguistic term. Specially, s_{-l} and s_l are the lower and upper limits, respectively, l is a positive integer, and S' must have the following characteristics [5,38]:

(1) The set is ordered: $s_\alpha\geq s_\beta$ if $\alpha\geq\beta$;
(2) There is the negation operator: $neg(s_\alpha)=s_{-\alpha}$, especially, $neg(s_0)=s_0$;
(3) Max operator: $max(s_\alpha,s_\beta)=s_\alpha$ if $s_\alpha\geq s_\beta$;
(4) Min operator: $min(s_\alpha,s_\beta)=s_\alpha$ if $s_\alpha\leq s_\beta$.

For example, a set of nine terms S' could be

$$S'=\{s_{-4}=extrimly\ poor,\ s_{-3}=very\ poor,\ s_{-2}=slightly\ poor,\ s_{-1}=poor,\ s_0=fair,\ s_1=slightly\ good,\ s_2=good,\ s_3=very\ good,\ s_4=extrimly\ good\}.$$

Analogously, the discrete term set S' can be also extended to a continuous term set [10] $\overline{S'}=\{\alpha\in[-l,l]\}$, whose elements also meet all the characteristics above. Correspondingly, if $s_\alpha\in S'$, we call s_α the original term and α the original term index; otherwise, we call s_α the virtual term and α the virtual term index.

3. Evaluation of the results of linguistic MAGDM based on term indices

In the real world, we may encounter a variety of GDM problems. So many different decision analysis methods are developed to deal with various GDM problems. In Section 1, we have introduced some commonly used methods for GDM. However, when we use different decision analysis methods to deal with the same GDM problem, we usually can not get consistent decision results. Even if we use the same decision analysis method to deal with the same GDM problem, decision results still vary with different weights of the attributes or the DMs during each decision-making process. Therefore, it necessitates a decision-making index system to evaluate the effect of the GDM results. By this way, we have a guide line to select a more appropriate decision analysis method or to set more reasonable weights of the attributes and DMs. In addition, in dynamic or interactive MAGDM, we also need to evaluate the decision-making effect of each DM, based on which we can adjust the weights of DMs.

In MAGDM with linguistic information, if the individual overall preference values and the collective overall preference values are expressed in form of linguistic terms, in order to simplify the representation and facilitate the calculation, we need do some preprocessings on these linguistic terms, that is, transforming the linguistic terms into corresponding term indices. In what follows the conversion functions are given to finish this transformation.

Definition 1. Let $\overline{S}=\{s_\alpha|\alpha\in[0,l]\}$ be a set of extended continuous linguistic terms, where l is an even number, $s_\alpha\in\overline{S}$ is a linguistic term, then the corresponding term index α can be got by the function I as follows:

$$I:\overline{S}\rightarrow[0,l],$$

$$I(s_\alpha)=\alpha, s_\alpha\in\overline{S}. \tag{1}$$

Definition 2. Let $\overline{S'}=\{s_\alpha|\alpha\in[-l,l]\}$ be a set of extended continuous linguistic terms, where l is a positive integer, $s_\alpha\in\overline{S'}$ is a linguistic term, then the corresponding term index α can be got by the following function I' :

$$I':\overline{S'}\rightarrow[-l,l],$$

$$I'(s_\alpha)=\alpha, s_\alpha\in\overline{S'}. \tag{2}$$

To facilitate the discussion, in this paper, let z_i denote the collective overall preference value of alternative x_i (which takes the form of linguistic term), α_i denote the corresponding term index, that is, $\alpha_i=I(z_i)$ if $z_i\in\overline{S}$ or $\alpha_i=I'(z_i)$ if $z_i\in\overline{S'}$; let z^k_i denote the individual overall preference value of alternative x_i given by the DM d_k (which takes the form of linguistic term), α^k_i denote the corresponding term index, that is, $\alpha^k_i=I(z^k_i)$ if $z^k_i\in\overline{S}$ or $\alpha^k_i=I'(z^k_i)$ if $z^k_i\in\overline{S'}$.

3.1. Evaluation of the decision-making effect of single decision maker

In order to quantify the level of the decision-making effect of each DM in linguistic MAGDM, here we compare the individual overall preference values with the collective overall preference values based on term indices from consistency, closeness and uniformity three aspects.

3.1.1. Consistency

For a given set of alternatives X, the degree of consistency of the individual overall preference values with respect to the collective overall preference values in rank can reflect the decision-making effect of DM to some extent. For this reason, this section defines the concept of consistency of the individual overall preference values given by the DM d_k with respect to the collective overall preference values by borrowing ideas from consistency of an ordered decision table in rough set theory [61].

In the original rough set theory introduced by Pawlak [62,63], the notion of consistency degree [62] is defined for a decision table, which in some sense could be regarded as measures for evaluating the decision performance of a decision table. Nevertheless, the consistency degree of a decision table cannot give elaborate depictions of the consistency for a given decision table. Therefore, Qian and Liang [64] introduced the consistency measures to assess the consistencies of a target concept and a decision table. Furthermore, they proposed three new measures to assess the entire decision performance of a decision-rule set extracted from a complete/incomplete decision table [65,66]. However, the original rough set theory does not consider attributes with preference-ordered domains, that is, criteria. Greco et al. [67,68] proposed the dominance-based rough sets approach (DRSA) to take into account the ordering properties of criteria. This innovation is mainly realized by substituting a dominance relation for the indiscernibility relation. Based on DRSA a consistency measure to calculate the consistency of an ordered decision table is also introduced by Qian and Liang [61].

In a given MAGDM problem with linguistic information, for $\forall x_i, x_h \in X$ $(1 \le i,h \le n,\ i \ne h)$, we say that x_h dominates x_i with respect to the individual overall preference values given by the DM d_k if $\alpha_h^k \ge \alpha_i^k$, and denoted by $x_h R_{\alpha^k}^{\ge} x_i$. That is, the dominance relation $R_{\alpha^k}^{\ge} = \{(x_h,x_i) \in X \times X \mid \alpha_h^k \ge \alpha_i^k\}$. The dominance class of alternative x_i with respect to the individual overall preference values given by the DM d_k is denoted by $[x_i]_{\alpha^k}^{\ge} = \{x_h \in X \mid \alpha_h^k \ge \alpha_i^k\}$. Let $X/R_{\alpha^k}^{\ge} = \{[x_1]_{\alpha^k}^{\ge}, [x_2]_{\alpha^k}^{\ge}, \ldots, [x_n]_{\alpha^k}^{\ge}\}$ denote the family set of the dominance classes derived by the individual overall preference values given by the DM d_k. Dominance classes in $X/R_{\alpha^k}^{\ge}$ do not constitute a partition of X in general, they constitute a covering of X.

Similarly, for $\forall x_i, x_h \in X$ $(1 \le i,h \le n, i \ne h)$, we say that x_h dominates x_i with respect to the collective overall preference values if $\alpha_h \ge \alpha_i$, and denoted by $x_h R_{\alpha}^{\ge} x_i$. That is, the dominance relation $R_{\alpha}^{\ge} = \{(x_h,x_i) \in X \times X \mid \alpha_h \ge \alpha_i\}$. The dominance class of alternative x_i with respect to the collective overall preference values is denoted by $[x_i]_{\alpha}^{\ge} = \{x_h \in X \mid \alpha_h \ge \alpha_i\}$. Let $X/R_{\alpha}^{\ge} = \{[x_1]_{\alpha}^{\ge}, [x_2]_{\alpha}^{\ge}, \ldots, [x_n]_{\alpha}^{\ge}\}$ denote the family set of the dominance classes derived by the collective overall preference values. Dominance classes in $X/R_{\alpha}^{\ge}$ do not constitute a partition of X in general, they constitute a covering of X.

Example 1. We select the example given by Ref. [55] to explain the concepts of the dominance class and the set of the dominance classes mentioned above. In this example, the individual overall preference values and the collective overall preference values which take the form of term indices are shown in Table 1.

Table 1
The individual overall preference values and the collective overall preference values given by Ref. [55].

Alternatives	Decision makers			
	d_1	d_2	d_3	Group
x_1	4.354	3.820	4.020	3.831
x_2	4.354	3.850	4.010	3.944
x_3	4.522	4.570	5.090	4.656
x_4	3.486	3.560	3.330	3.358

In this table, the collective overall preference value of each alternative x_i can be denoted as follows: $\alpha_1 = 3.831$, $\alpha_2 = 3.944$, $\alpha_3 = 4.656$, $\alpha_4 = 3.358$; and the individual overall preference value of each alternative x_i given by the DM d_1 can be denoted as follows: $\alpha_1^1 = 4.354$, $\alpha_2^1 = 4.354$, $\alpha_3^1 = 4.522$, $\alpha_4^1 = 3.486$.

From Table 1, one can obtain that the dominance class of each alternative x_i with respect to the individual overall preference values given by the DM d_1 are

$$[x_1]_{\alpha^1}^{\ge} = \{x_1,x_2,x_3\}, \quad [x_2]_{\alpha^1}^{\ge} = \{x_1,x_2,x_3\},$$

$$[x_3]_{\alpha^1}^{\ge} = \{x_3\}, \quad [x_4]_{\alpha^1}^{\ge} = \{x_1,x_2,x_3,x_4\},$$

and the set of the dominance classes derived by the individual overall preference values given by the DM d_1 are

$$\begin{aligned} X/R_{\alpha^1}^{\ge} &= \{[x_1]_{\alpha^1}^{\ge}, [x_2]_{\alpha^1}^{\ge}, [x_3]_{\alpha^1}^{\ge}, [x_4]_{\alpha^1}^{\ge}\} \\ &= \{\{x_1,x_2,x_3\},\{x_1,x_2,x_3\},\{x_3\},\{x_1,x_2,x_3,x_4\}\}. \end{aligned}$$

Analogously, one can obtain that the dominance class of each alternative x_i with respect to the collective overall preference values are

$$[x_1]_{\alpha}^{\ge} = \{x_1,x_2,x_3\}, \quad [x_2]_{\alpha}^{\ge} = \{x_2,x_3\}, \quad [x_3]_{\alpha}^{\ge} = \{x_3\}, \quad [x_4]_{\alpha}^{\ge} = \{x_1,x_2,x_3,x_4\},$$

and the dominance classes derived by the collective overall preference values are

$$\begin{aligned} X/R_{\alpha}^{\ge} &= \{[x_1]_{\alpha}^{\ge}, [x_2]_{\alpha}^{\ge}, [x_3]_{\alpha}^{\ge}, [x_4]_{\alpha}^{\ge}\} \\ &= \{\{x_1,x_2,x_3\},\{x_2,x_3\},\{x_3\},\{x_1,x_2,x_3,x_4\}\}. \end{aligned}$$

Definition 3. In a given multi-attribute group decision-making with linguistic information, the consistency of the individual overall preference values given by the DM d_k with respect to the collective overall preference values is defined as

$$C^k = \frac{1}{n} \sum_{x_i \in X} \frac{|[x_i]_{\alpha^k}^{\ge} \bigcap [x_i]_{\alpha}^{\ge}|}{|[x_i]_{\alpha^k}^{\ge}|}, \tag{3}$$

where $X = \{x_1, x_2, \ldots, x_n\}$ is the set of alternatives, $1 \le i \le n$, $[x_i]_{\alpha^k}^{\ge} \in X/R_{\alpha^k}^{\ge}$ is the dominance class of alternative x_i with respect to the individual overall preference values given by the DM d_k, $[x_i]_{\alpha}^{\ge} \in X/R_{\alpha}^{\ge}$ is the dominance class of alternative x_i with respect to the collective overall preference values.

Example 2. From Definition 3, one can obtain the consistency C^1 of the individual overall preference values given by the DM d_1 with respect to the collective overall preference values on the basis of Example 1, that is

$$C^1 = \frac{1}{4} \sum_{x_i \in X} \frac{|[x_i]_{\alpha^1}^{\ge} \bigcap [x_i]_{\alpha}^{\ge}|}{|[x_i]_{\alpha^1}^{\ge}|} = \frac{1}{4}\left(\frac{3}{3} + \frac{2}{3} + \frac{1}{1} + \frac{4}{4}\right) = \frac{11}{12} = 0.9167.$$

Property 1. $0 < C^k \le 1$.

The consistency of the individual overall preference values given by the DM d_k with respect to the collective overall preference values depends closely on the value of C^k. That is, C^k closer to 0, the poorer the consistency; oppositely, C^k closer to 1, the better the consistency.

If $C^k=1$, the individual overall preference values given by the DM d_k can be said to be consistent with respect to the collective overall preference values, that is, for $\forall x_i \in X$, one has that $[x_i]^{\geq}_{\alpha^k} \subseteq [x_i]^{\geq}_{\alpha}$.

3.1.2. Closeness

In this section, we will analyze the decision-making effect of DM from another point of view, that is, by comparing the individual overall preference values with the collective overall preference values from the perspective of the size of value, we can give the definition of closeness of the individual overall preference values with respect to the collective overall preference values.

Definition 4. In a given multi-attribute group decision-making with linguistic information, the absolute difference between α_i^k (i.e., the individual overall preference value of alternative x_i given by the DM d_k) and α_i (i.e., the collective overall preference value of alternative x_i) is defined as

$$\Delta_i^k = |\alpha_i^k - \alpha_i|. \tag{4}$$

Property 2. *$0 \leq \Delta_i^k \leq N$, where the values of N have the following two cases: (1) If the discrete term set $S=\{s_\alpha | \alpha=0,1,\ldots,l\}$, then $N=l$; (2) If the discrete term set $S'=\{s_\alpha | \alpha=-l,\ldots,-1,0,1,\ldots,l\}$, then $N=2l$.*

Definition 5. In a given multi-attribute group decision-making with linguistic information, the closeness of the individual overall preference values given by the DM d_k with respect to the collective overall preference values is defined as

$$T^k = \frac{\sqrt{\sum_{i=1}^{n}(\Delta_i^k - N)^2}}{\sqrt{\sum_{i=1}^{n}(\Delta_i^k - N)^2} + \sqrt{\sum_{i=1}^{n}(\Delta_i^k)^2}}, \tag{5}$$

where Δ_i^k is the absolute difference between α_i^k and α_i $(1 \leq i \leq n)$, N is the maximum value that the absolute difference could be, i.e., $N=l$ or $N=2l$, which is described in detail in Property 2.

Example 3 (*Continued from Example 1*). From Definition 4, one can obtain the absolute difference Δ_i^1 between α_i^1 (i.e., the individual overall preference value of alternative x_i given by the DM d_1) and α_i (i.e., the collective overall preference value of alternative x_i), that is

$$\Delta_1^1 = |\alpha_1^1 - \alpha_1| = |4.354 - 3.831| = 0.523,$$
$$\Delta_2^1 = |\alpha_2^1 - \alpha_1| = |4.354 - 3.944| = 0.41, \quad \Delta_3^1 = 0.134, \quad \Delta_4^1 = 0.128.$$

Since the discrete term set used in the example given by Ref. [55] is $S=\{s_\alpha | \alpha=0,1,\ldots,6\}$, according to Property 2, one has that $N=6$. Then one can obtain the closeness T^1 of the individual overall preference values given by the DM d_1 with respect to the collective overall preference values from Definition 5, that is,

$$T^1 = \frac{\sqrt{(0.523-6)^2+(0.41-6)^2+(0.134-6)^2+(0.128-6)^2}}{\sqrt{(0.523-6)^2+(0.41-6)^2+(0.134-6)^2+(0.128-6)^2}+\sqrt{0.523^2+0.41^2+0.134^2+0.128^2}} = 0.943.$$

Property 3. $0 \leq T^k \leq 1$.

If $T^k=0$, then for each alternative $x_i \in X$ $(1 \leq i \leq n)$, $\Delta_i^k = N$ holds, i.e., the individual overall preference values given by the DM d_k are the least close to the collective overall preference values; if $T^k=1$, then for each alternative $x_i \in X$ $(1 \leq i \leq n)$, $\Delta_i^k=0$ holds, i.e., the individual overall preference values given by the DM d_k are the most close to the collective overall preference values (i.e., both of them are the same).

3.1.3. Uniformity

Next, we use the concept of entropy to quantify the distribution of the absolute difference Δ_i^k $(1 \leq i \leq n)$, and then provide the definition of uniformity to measure the degree of deviation of the individual overall preference values from the collective overall preference values.

Definition 6. In a given multi-attribute group decision-making with linguistic information, the uniformity of the individual overall preference values given by the DM d_k with respect to the collective overall preference values is defined as

$$U^k = -\frac{1}{\log_2^n}\sum_{i=1}^{n}\frac{\Delta_i^k}{\sum_{i=1}^{n}\Delta_i^k}\log_2\frac{\Delta_i^k}{\sum_{i=1}^{n}\Delta_i^k}, \tag{6}$$

where Δ_i^k is the absolute difference between α_i^k and α_i $(1 \leq i \leq n)$, n is the number of the alternatives.

Example 4. From Example 3, one can obtain that

$$\sum_{i=1}^{4}\Delta_i^1 = 0.523+0.41+0.134+0.128 = 1.195.$$

Then from Definition 6, one can obtain the uniformity U^1 of the individual overall preference values given by the DM d_1 with respect to the collective overall preference values, that is,

$$\begin{aligned} U^1 &= -\frac{1}{\log_2^4}\sum_{i=1}^{4}\frac{\Delta_i^1}{\sum_{i=1}^{4}\Delta_i^1}\log_2\frac{\Delta_i^1}{\sum_{i=1}^{4}\Delta_i^1} \\ &= -\frac{1}{2}\left(\frac{0.523}{1.195}\log_2\frac{0.523}{1.195}+\frac{0.41}{1.195}\log_2\frac{0.41}{1.195}\right. \\ &\quad \left. +\frac{0.134}{1.195}\log_2\frac{0.134}{1.195}+\frac{0.128}{1.195}\log_2\frac{0.128}{1.195}\right) \\ &= 0.8752. \end{aligned}$$

Property 4. $0 \leq U^k \leq 1$.

U^k closer to 0, the more nonuniform the distribution of Δ_i^k $(1 \leq i \leq n)$; otherwise, U^k closer to 1, the more uniform the distribution of Δ_i^k $(1 \leq i \leq n)$.

If $U^k=1$, it shows that the absolute difference Δ_i^k $(1 \leq i \leq n)$ is the uniform distribution.

Specially, if for $\forall i$ $(1 \leq i \leq n)$, one has $\Delta_i^k=0$, i.e., $\sum_{i=1}^{n}\Delta_i^k=0$, then we believe that the absolute difference is the uniform distribution, and prescribe $U^k=1$.

Based on the above three indices, we can evaluate and compare the decision-making effect of all DMs. Since the aim of the linguistic MAGDM is usually to rank all the alternatives or to select the best one, the consistency index can be viewed as the most important evaluation index. So we can propose the following evaluation criteria: for the same linguistic MAGDM problem, the decision results of the DM with higher consistency are closer to the group decision results; if there are two different DMs with the same consistency, the decision results of the one with higher closeness are closer to the group decision results; then if there are still two different DMs with the same closeness, the decision results of the one with higher uniformity are closer to the group decision results. Based on the above criteria, we can rank all DMs.

3.2. Evaluation of the effect of group decision-making

For a given set of decision makers $D=\{d_1,d_2,\dots,d_t\}$, and their weight vector $v=\{v_1,v_2,\dots,v_t\}^T$ where $v_k \ge 0$ and $\sum_{k=1}^t v_k=1$, we can use the given weights of DMs to get the weighted average of the values on each index of each DM respectively, and accordingly we can define three total indices, namely, total consistency, total closeness and total uniformity, to reflect the effect of GDM.

Definition 7. In a given multi-attribute group decision-making with linguistic information, the total consistency C is defined as

$$C=\sum_{k=1}^{t} v_k C^k, \tag{7}$$

where v_k denotes the weight of the DM d_k and C^k denotes the consistency of the individual overall preference values given by the DM d_k with respect to the collective overall preference values.

Definition 8. In a given multi-attribute group decision-making with linguistic information, the total closeness T is defined as

$$T=\sum_{k=1}^{t} v_k T^k, \tag{8}$$

where v_k denotes the weight of the DM d_k and T^k denotes the closeness of the individual overall preference values given by the DM d_k with respect to the collective overall preference values.

Definition 9. In a given multi-attribute group decision-making with linguistic information, the total uniformity U is defined as

$$U=\sum_{k=1}^{t} v_k U^k, \tag{9}$$

where v_k denotes the weight of the DM d_k and U^k denotes the uniformity of the individual overall preference values given by the DM d_k with respect to the collective overall preference values.

Obviously, one can easily notice that the three properties $0<C\le 1$, $0\le T\le 1$ and $0\le U\le 1$ hold true.

Example 5 (*Continued from Examples 1 and 2*). In Example 2, one has obtained $C^1=0.9167$; Analogously, one can obtain $C^2=1$, and $C^3=0.9167$.

Since the weights of DMs are determined clearly in the example given by Ref. [55](see Table 2), according to Definition 7, one can obtain that

$$C=\sum_{k=1}^{3} v_k C^k = 0.4{*}0.9167+0.3{*}1+0.3{*}0.9167=0.9417.$$

The three total indices C, T and U reflect the effect of GDM from three different sides and have some reference value for evaluating and comparing the effect of different GDM methods. Because the aim of the linguistic MAGDM is usually to rank all the alternatives or to select the best one, the total consistency index can be viewed as the most important evaluation index. So we propose the following evaluation criteria for GDM: for the same linguistic MAGDM problem, if we use different MAGDM methods, the method with the higher total consistency shows that the weighted average of the decision results of all DMs are closer to the group decision results; if there are two different methods with the same total consistency, the one with the higher total closeness shows that the weighted average of the decision results of all DMs are closer to the group decision results; then if there are still two different methods with the same total closeness, the one with higher total uniformity shows that the weighted average of the decision results of all DMs are closer to the group decision results. Based on the above criteria, we can compare the effect of different linguistic MAGDM methods.

Table 2
The weights of DMs given by Ref. [55].

d_1	d_2	d_3
0.4	0.3	0.3

4. Illustrative example

In order to illustrate the practicality and effectiveness of the method proposed above, we select two typical examples of MAGDM with linguistic information from Refs. [55,38]. Although the decision analysis methods used in the two examples are different, the method proposed in this paper are applicable to both of them. Because the weights of DMs and the collective overall preference values have been obtained in Refs. [55] and [38] respectively, and the individual overall preference values in Ref. [55] also have been provided clearly, though the individual overall preference values in Ref. [38] have not been directly given, we can simply calculate the values by using the optimal weight vector of the attribute and the corresponding equation provided by the paper (see, Tables 1-4).

It is noteworthy that the discrete term set used in the two examples aree different. The discrete term set used in the example given by Ref. [55] is $S=\{s_\alpha|\alpha=0,1,\dots,6\}$, according to property 2, one has that N=6; However, the discrete term set used in the example given by Ref. [38] is $S'=\{s_\alpha|\alpha=-4,\dots,\ -1,0,1,\dots,4\}$, according to Property 2, one has that N=8. Next, we use the evaluation method proposed in this paper to analyze both the decision-making effect of each DM and the effect of GDM in the two examples respectively. The results are shown in detail in Tables 5 and 6.

In Table 5, we can find that the consistency of the DM d_2 is the highest one in the all DMs, so the decision results of d_2 is closest to the group decision results; then for the DMs d_1 and d_3, we notice that the consistency of both of them is the same, but the closeness of d_3 is higher than d_1, so the decision results of d_3 is closer to the group decision results than d_1. Thus, we can rank all DMs as $d_2>d_3>d_1$. Similarly, in Table 6, we can also rank all DMs as $d_1>d_3>d_2$. In a word, according to the consistency, closeness and uniformity, we can give the ranking results of the

Table 3
The individual overall preference values and the collective overall preference values given by Ref. [38].

Alternatives	Decision makers			
	d_1	d_2	d_3	Group
x_1	1.8677	2.5624	2.173	2.2011
x_2	0.9015	0.9447	0.565	0.8037
x_3	2.6899	2.44	2.0962	2.4087
x_4	1.0216	−0.0769	0.0889	0.3446

Table 4
The weights of DMs given by Ref. [38].

d_1	d_2	d_3
1/3	1/3	1/3

Table 5
The evaluation results on the example given by Ref. [55].

Decision makers	Evaluation indices		
	Consistency	Closeness	Uniformity
d_1	0.9167	0.9430	0.8752
d_2	1	0.9801	0.8056
d_3	0.9167	0.9605	0.7226
Group	0.9417	0.9594	0.8085

Table 6
The evaluation results on the example given by Ref. [38].

Decision makers	Evaluation indices		
	Consistency	Closeness	Uniformity
d_1	0.9375	0.9497	0.8678
d_2	0.875	0.9644	0.8102
d_3	0.875	0.9707	0.8674
Group	0.8958	0.9616	0.8485

decision-making effect of all DMs, which can help us to adjust the weights of DMs in the next decision-making process.

At the same time, the total consistency, total closeness and total uniformity of the two examples are also shown in Tables 5 and 6 respectively, from which we are able to evaluate and analyze the effect of GDM. If we analyze the example illustrated in Ref. [55] or Ref. [38] by another decision method or choosing different weights of the attributes or the DMs, straightforwardly different ranking results of all the alternatives will be obtained. Then, according to the total consistency, total closeness and total uniformity, we can compare the effect of different GDM methods and it will help us to decide how to select the appropriate weights and method to make better decision.

5. Conclusions

At present, linguistic MAGDM methods are widely and well studied in theory, however, many more practical aspects, such as how to evaluate and compare the effect of GDM, how to reflect individual preference in collective preference as well as how to determine the extent an individual preference reflected, are relatively open and less studied.

In this paper, based on the existing linguistic MAGDM methods we define three key indices, i.e., consistency, closeness and uniformity, to evaluate the decision-making effect of DM from three different aspects by comparing the individual overall preference values with the collective ones. Correspondingly, by using the given weights of DMs to get the weighted average of the values on each index of each DM respectively, we also define the concepts of the total consistency, total closeness and total uniformity to evaluate the effect of GDM. In this way, it is feasible to not only judge the decision-making effect of DM, but also reflect the effect of GDM to a certain extent. The practicality and effectiveness of the proposed method are illustrated by two typical examples. These results will be helpful for further study of dynamic or interactive GDM.

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